The Minimum Area Discrepancy Method (MADM): A New Tool to Correlate Force Displacement Responses.

The development of computing capability has led to an ever-increasing use of numerical modelling in science and engineering. It is essential to validate any numerical model in order to ensure credibility of the results. Usually, the response of a model is compared to that of the represented system for a set of (physical) experimental test configurations. Physical testing rely on instrumentation most often in the form of accelerometers or force transducers which measure accelerations and forces respectively. These outputs can in turn be post-processed and compared against a set of engineering criteria. Readings from transducers as a function of time can be correlated using a CORA rating. Others, like force vs deflection responses, usually cannot be used in CORA due to the fact that functional values (Y-axis) are not unique with respect to X-axis values. As an example a force reading of 1.5kN (Y-value) occurs for multiple displacement (X-axis) values, as illustrated in Figure 1.

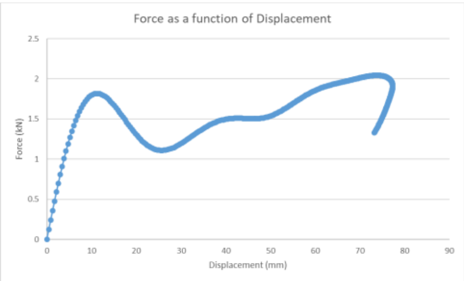


Figure 1: Typical Force vs Deflection curve

The MADM introduces a method to calculate a correlation coefficient for "typical" force-displacement signals in order to rate how close they are to test response. The MADM correlation criteria utilises an area method and aims to maximise area overlap; a complete (maximum) overlap indicates perfect correlation.

Figure 2 contains an example of numerical analysis results (labelled "model") which can be overlaid on the test data represented by the "upper" "lower" and "average" test-corridors.

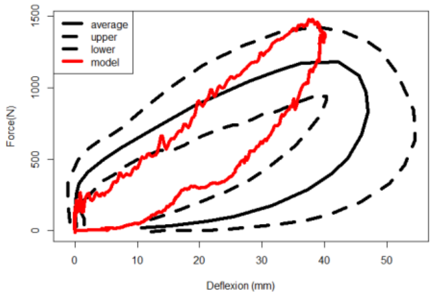
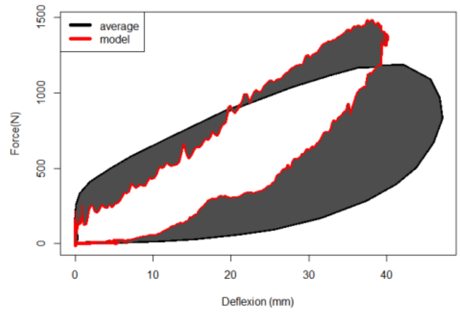


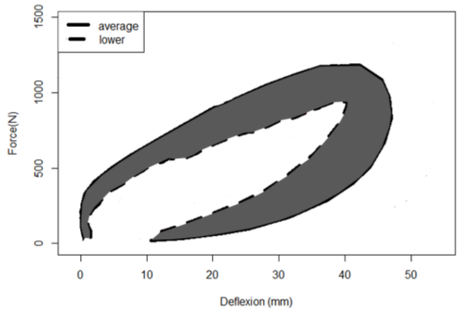
Figure 2: Typical CAE response against test corridors

Based on Figure 2 three areas can be defined:

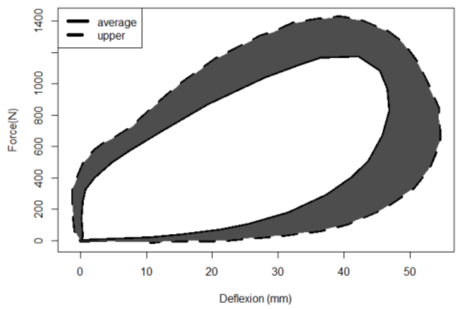
- A_{model}: area between the *model* and the *average* corridor.



- A_{lower}: area between the *lower* corridor and the *average* corridor



- A_{upper}: area between the *upper* corridor and the *average* corridor



Using these definitions MADM can now be introduced using the following steps. Firstly the ratio 'R' can be calculated as Equation 1:

$$R = \frac{A_{model}}{\frac{(A_{upper} + A_{lower})}{2}}$$

Equation 1: Calculation of R: area ratio overlap between the CAE model and the test corridors

'R' represents the level of discrepancy between the numerical model and the experimental test data using the average, lower and upper test corridors. The ratio 'R' is then normalised and referred to as the MADM correlation number, as per Equation 2, which is subsequently adjusted to suit the desired degree of correlation, by changing the values of n and m from Equation 2.

$$MADM_{n,m} = \frac{1}{1 + nR^m}$$

Equation 2: MADM Generic Form

The effect of the n and m in Equation 2MADM correlation progression is illustrated in Figure 3

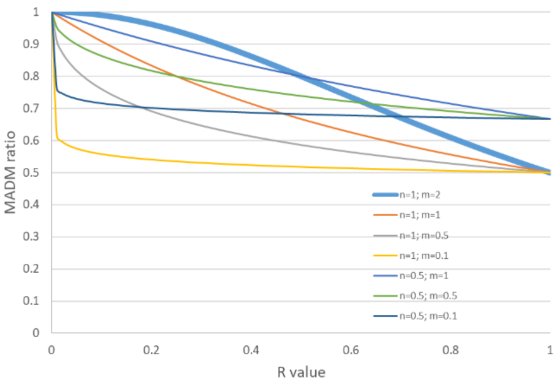


Figure 3: Relationship between the MADM ratio and the parameters n and m.

From previous studies undertaken, the authors suggest n and m values of 1 and 2 respectively, leading to the MADM correlation function shown in Equation 3.

$$MADM_{1,2} = \frac{1}{1 + R^2}$$

Equation 3: Recommended values n and m values for MADM in the case of impact biomechanics

The MADM has been tested in the development of a thorax spring model assembly [1][2] for which the springs characteristics had to be optimised for two loadcases (low impact energy and high impact energy).

Low impact Energy:

- $MADM_{1,2}$ (initial) = 0.79
- $MADM_{1,2}$ (final) = 0.93

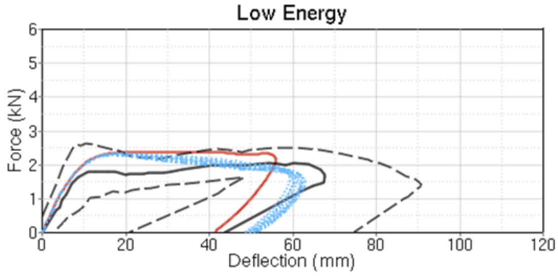


Figure 4: Low Impact Energy Scenario (Initial CAE response - RED; Optimised Response MADM - BLUE))

High Energy:

- $MADM_{1,2}$ (initial) = 0.89
- $MADM_{1,2}$ (final) = 0.92

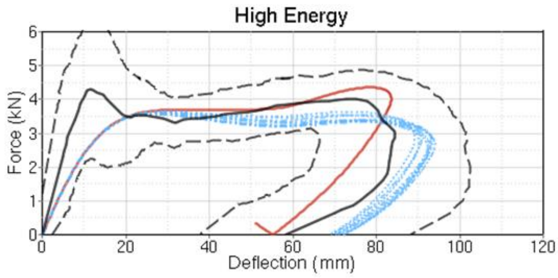


Figure 5: High Impact Energy Scenario (Initial CAE response - RED; Optimised Response MADM - BLUE))

References:

[1] Peres, J., Bastien, C., Christensen, J., Asghapour, Z., 2019 **"A minimum area discrepancy method (MADM) for force displacement response correlation"**, Computer Methods in Biomechanics and Biomedical Engineering, Volume 22, 2019 – Issue 11.

[2] Peres, J., Bastien, C., Christensen, J., 2018 "**A New Correlation Method to Analyse the Accuracy of CAE models in Force Displacement Response Scenarios – Application to a HBM Thorax Impact**", CARHS 2018 conference, Berlin (Germany)

The programme MADM is available free of charge by contacting:

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